EFFECT OF FRP REINFORCEMENT ON ARCHING ACTION IN FRP-STRENGTHENED CONTINUOUS CONCRETE BEAMS

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Abstract

Compressive membrane action (CMA) has been proven to be favorable for the enhancement of the load bearing capacity of longitudinally restrained concrete beams. To investigate the effect of Fibre Reinforced Polymer (FRP) applications on CMA in FRP-strengthened concrete beams, a rigid-plastic failure mechanism and a strength prediction model are employed. The debonding and fracture mechanisms of the FRP failure are considered in a theoretical analysis. Effects of parameters such as the cross-section area, modulus of elastisity and ultimate strain of the FRP reinforcement are examined by defining an enhancement factor to measure the influence of CMA on the load capacity of FRP-strengthened concrete beams. The results show in a quantative way how the enhancement factor increases with increasing area, modulus of elasticity and the beneficial effect of CMA decreases when such parameters increase. Results also indicate that the effect of the ultimate strain of the FRP reinforcement is negligible.

Keywords: Compressive membrane action, Reinforced concrete, Fibre reinforced polymers, Enhancement factor, Beams, Parameter study

1 Introduction

Since compressive membrane action (CMA) has been recognized for laterally restrained members, many research has been performed on this topic particularly in the field of reinforced concrete (RC) slabs (Taylor, Rankin & Cleland 2002). Research results have shown that CMA is beneficial for strength enhancement. With regard to the investigation of CMA in concrete members, one commonly applied method proposed by Park & Gamble (2000), is using the plastic theory to obtain member's resistance. This method considers CMA to be initially associated with the bending capacity and is good at estimating the CMA capacity of laterally restrained RC members. With the increased application of Fibre Reinforced Polymer (FRP) for the strengthening of concrete structures, a lot of research literature is available on the effect of FRP on the strength enhancement while comparing to the standard concrete structures (Kai & Li 2012). Accordingly, regarding the concept of CMA, it is desirable to investigate how the CMA is affected when fibre reinforcement is taken into accounted.

In this paper, the perfectly rigid plastic theory was employed to account for the effect of FRP on the load bearing capacity of RC members considering CMA. Specifically, a general perfectly rigid-plastic failure mechanism and a commonly applied strength prediction model regarding CMA are employed to investigate the influence of CMA on the load bearing capacity. Consequently, the effect of FRP on CMA in FRP-strengthened RC beams was investigated by parametric study with parameters such as the cross-sectional area, elastic modulus and ultimate strain of the FRP.

2 Prediction model

2.1 Assumptions

A model with four idealized plastic hinges formed symmetrically along the beam is chosen to represent a FRP-strengthened RC beam. A perfectly rigid plastic mechanism is basically assumed. A complete symmetry along the span is assumed with respect to geometry, reinforcement, loading, boundary conditions and deformations. The lateral restraints are idealized to be equivalent axial springs with stiffness K_a. Fig. 1 shows the schematic view of the model, where β is the ratio of the span length l_n from the plastic hinge at the beam end to the nearest hinge in the span to the beam span l.

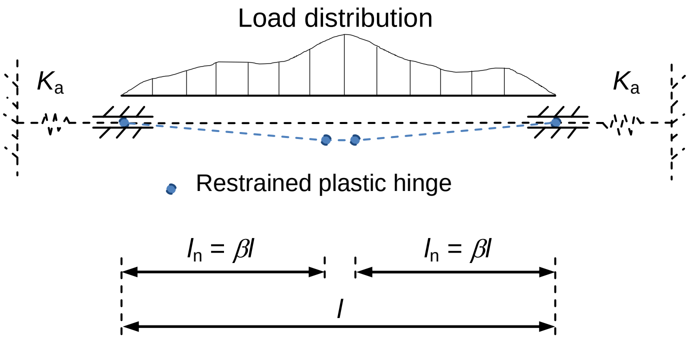


Fig. 1 Schematic view of laterally restrained beam

To calculate the sectional moments and forces, the commonly-adopted assumptions are used, including the assumptions of plane-section, a full composite action of FRP, an idealized equivalent rectangular stress block for compressive concrete, an ignorance of concrete tensile strength and the compressive strength of FRP, a bilinear stress-strain relationship for steel bars and a linear stress-strain relationship for FRP (*fib* 2001). Note that the full composite action of FRP means that the beam failure occurs when the tensile steel reinforcement has yielded followed by concrete crushing whereas the FRP remains intact. And for generality, it is believed that the FRP can be placed at both the bottom of the mid-span and the top of the beam ends.Further, the axial force at the beam end (N_{u1}) is assumed to be equal to the axial force in the span (N_{u0}), as show in equation (1)

$$N = N_{u0} = N_{u1} \tag{1}$$

Besides, the compressive strain of the beam is assumed to be distributed uniformly along the beam span with a value of $\varepsilon = N/(E_c A_c)$, where N is the axial force, E_c is elastic modulus of concrete and A_c is the cross-sectional area.

2.1 Formulations and verification

Due to symmetry, one-half of the deformed shape is shown in Fig. 2. The axial force along the beam causes a horizontal displacement at the support, $t = N/K_a$. Given the assumption of the uniform compression strain along the beam, the contraction of the portion in Fig. 2 is $\varepsilon\beta l$, and the contraction of the middle portion is $(1\text{-}2\beta)\varepsilon l$. For a given beam deflection in the span δ, we have $\tan\theta \approx \delta/\beta l$ assuming that the rotation at the beam end θ is so small that $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ hold. Recalling the expressions of the beam strain ε and lateral restraint deformation t, the compatibility requirement can be expressed as in equation (2)

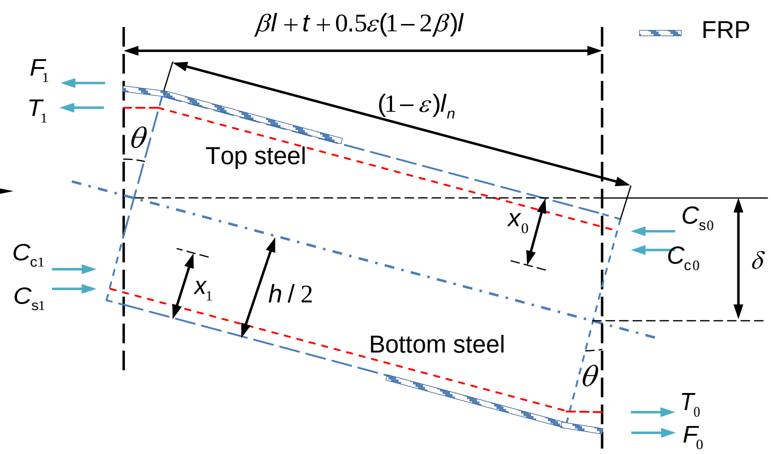


Fig. 2 Idealized deformation of one-half of the restrained beam

$$h - x_0 - x_1 = \frac{\delta}{2} + \frac{N\beta l^2}{2\delta}\left(\frac{1}{E_c A_c} + \frac{2}{K_a l}\right) \tag{2}$$

where x_0 and x_1 are the neutral-axis depth in the span and at the beam end, respectively; h is the beam depth. According to Fig. 2, equation (1) can be rewritten as

$$C_{c0} + C_{s0} - T_0 - F_0 = C_{c1} + C_{s1} - T_1 - F_1 \tag{3}$$

where C_{c0} and C_{c1} are the concrete compressive forces, C_{s0} and C_{s1} the steel compressive forces, T_0 and T_1 the steel tensile forces, and F_0 and F_1 the FRP tensile forces, acting on sections at the mid-span and the beam end, respectively.

Considering that all terms in equation (3) can be expressed in terms of given geometric and material properties and unknowns x_1 and x_0 based on a sectional force equilibrium analysis, x_0 and x_1 are obtained by solving the equations (2) and (3) simultaneously for a given δ. Further, the moments at the mid-span and the beam ends can be calculated and the load bearing capacity can be calculated. Repeating such procedures for different values of δ, a series of values of the load bearing capacity are obtained, and the maximum value can be seen as the ultimate load bearing capacity.

Note that, according to the assumption of the failure mode, the strain of the outer fibre of compressive concrete reaches to its ultimate strain, the tensile strain of steel is considerably larger than the yield strain, and the strain of FRP is less than its ultimate strain. In this paper, the ultimate concrete strain is 3.5‰ (*fib* 2001), the steel yield strain is calculated by its yield stress, and the ultimate strain of FRP is the test value or a value of 1.5% is selected if not specified. In addition, the lateral stiffness is calculated based on the actual stiffness due to the adjacent structural elements or, for simplicity, a relatively large stiffness, such as 1×10^6 kN/m can be selected if no such information can be obtained.

To verify the extensive model, a standard laterally restrained RC beam (A1) (Su, Tian & Song 2009) and a laterally restrained FRP-strengthened RC beam (FR-1) (Orton, Jirsa & Bayrak 2009) were selected, each of which can be idealized as the model shown in Fig. 1. The results show that the load bearing capacity is 181.8 kN and 146.6 kN, with an acceptable 8% and 1% overestimation compared to the tested results of 168.0 kN and 145.5 kN, for A1 and FR-1, respectively, which implies the feasibility and the effectiveness of the extended model for FRP-strengthened RC beams.

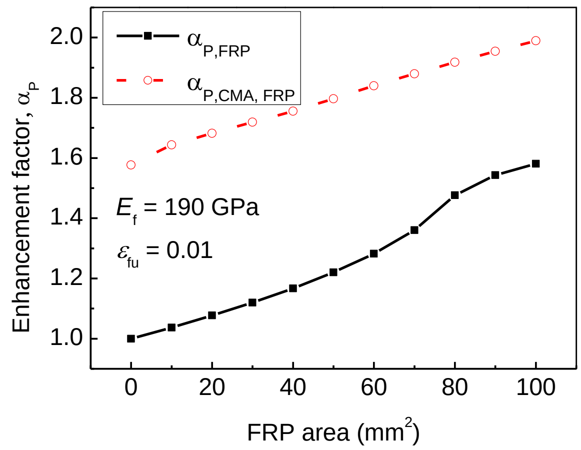
3 Effect of FRP reinforcements

Considering that CMA is affected by many factors and that both the FRP strengthening and CMA are beneficial to the enhancement of the beam strength, it is necessary to select a dimensionless quantity to measure the effect of CMA. Therefore, a strength enhancement factor α_P is adopted, and defined as $\alpha_{P,FRP} = P_{FRP}/P_0$ and $\alpha_{P,CMA,FRP} = P_{CMA,FRP}/P_0$, where $\alpha_{P,CMA}$ and $\alpha_{P,CMA,FRP}$ are enhancement factors considering the FRP enhancement and considering the enhancement of both FRP and CMA, respectively; P_{FRP} and $P_{CMA,FRP}$ are the peak resistance loads of a FRP-strengthened RC beam obtained by the common FRP calculation models (*fib* 2001) and the proposed method in this study, respectively; and P_0 is the peak resistance load of a regular RC beam calculated by perfectly rigid-plastic analysis without the consideration of CMA.

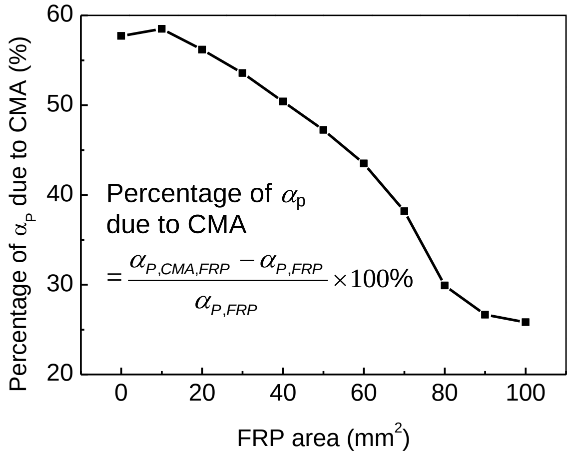
A beam similar to the four-point loaded two-span test beam (Vasseur 2009) is adopted as the benchmark beam in the comparison studies. This 10 m beam is assumed to be laterally restrained (K_a = 10^6 kN/m) at both ends with a 200 × 400 mm section and a value of 0.20 for β is selected. C30/37 concrete is used and three steel bars are initially placed along the beam length at both the beam top ($2\phi12+1\phi18$) and the beam bottom ($2\phi12+1\phi20$). The elastic modulus and the yield stress of the steel bars are 200 GPa and 500 MPa, respectively. The CFRP with a sectional area of 120 mm^2 is applied along the beam at tension zones and the elastic modulus and the ultimate strain of FRP are 190 GPa and 1%, respectively. All these values apply if no further information is indicated.

3.1 Effect of the sectional area of FRP

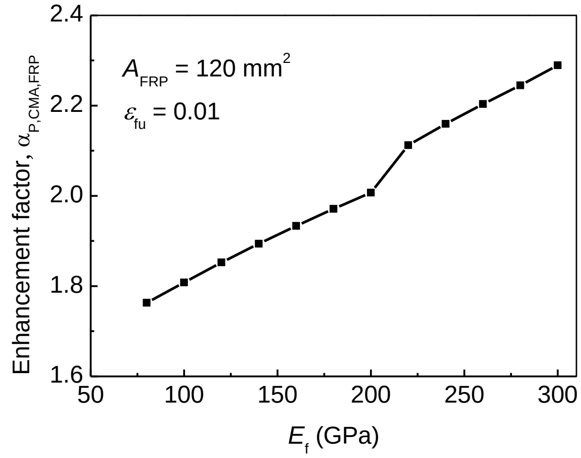
For laterally restrained FRP-strengthened RC beams, the vertical load resistances according to *fib* (2001) without the consideration of CMA and the vertical load resistances according to the proposed method in this study were calculated for different values of the FRP cross-sectional area. Then the enhancement factors for these two conditions were obtained, as shown in Fig. 3a. It indicates that the enhancement factor increases with increasing FRP reinforcement when CMA is considered. For instance, the enhancement factor increases by 16.7% when the sectional area of FRP is 40 mm^2 compared with the case that no FRP is applied. It is interesting to note that the percentage of α_P due to CMA, which is defined as the ratio of the difference between $\alpha_{P,FRP}$ and $\alpha_{P,CMA,\,FRP}$ to $\alpha_{P,FRP}$, decreases with increasing FRP reinforcement, as shown in Fig. 3b. In the case where the FRP area is 100 mm^2, the load resistance of the specimen gets a 58.1% increase when FRP is applied, and the specimen obtains an extra 40.8% increase in load resistance if CMA is considered. Therefore, the consideration of CMA for FRP-strengthened RC beams is favorable to structure design.



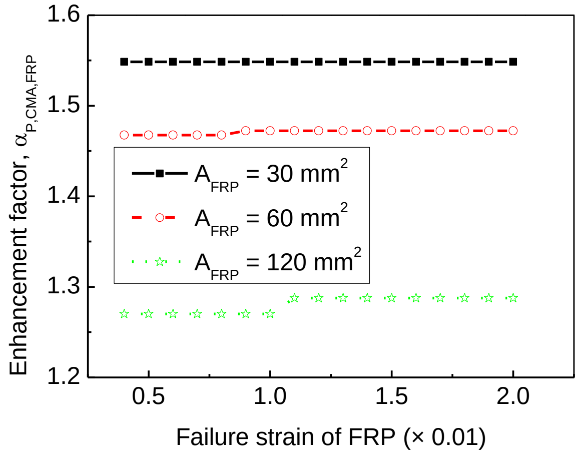
(a) Effect of FRP area



(b) Percentage of α_p due to CMA



(c) Effect of elastic modulus



(d) Effect of failure strain

Fig. 3 Effect of FRP reinforcement on CMA

3.2 Effect of the elastic modulus of FRP

Regarding the effect of the elastic modulus of FRP, a general range from 80 to 300 GPa for different kinds of fibres was selected. The enhancement factors are shown in Fig. 3c, which implies that a larger elastic modulus of fibre contributes to a larger load bearing capacity. The relationship between the elastic modulus of fibre and the beam resistance can be seen as a linear relationship, however, the increase of the enhancement factor or load bearing capacity is not as significant as the increase of the elastic modulus. For example, in the case of Fig. 3c, the load bearing capacity only increases 2% from 305.0 to 312.5 kN when the elastic modulus increases by 20% from 100 to 120 GPa.

3.3 Effect of the failure strain of FRP

It is well known that debonding failure and fracture failure may happen for FRP-strengthened concrete structures. Giving the fact that the FRP debonding strain differs a lot from the FRP fracture strain, it is necessary to investigate how the FRP failure strain affect CMA. According to Lander (2009), a choice of the debonding strain for CFRP is 0.4% and the nominal ultimate strain of FRP is generally less than 2%. However, 0.4% should be considered as a minimum value and much higher values are documented in literatures. Therefore, in this study, a minimum failure strain of 0.4% and a maximum strain of 2% were chosen to examine the effect of FRP failure strain, as shown in Fig. 3d. Further, different cross-sectional areas of FRP were also considered. Fig. 3d shows that the effect of the ultimate strain of FRP on the enhancement factor is found to be negligible, which means that the model proposed above is feasible for a FRP-strengthened RC beam whether a debonding failure and fracture failure occurs as long as it can be idealized to be the model shown in Fig. 1.

4 Conclusions

The Park and Gamble's method accounting for CMA was extended and validated for FRP-strengthened RC beams and the effect of FRP reinforcement on CMA was investigated in this paper. Following conclusions can be drawn:

(1) The extended model was feasible and effective in taking CMA into account to predict the ultimate load bearing capacity for FRP-strengthened RC beams;

(2) The effect of CMA increases with the increase of the sectional area of FRP. Meanwhile, the proportion of the enhancement of the load bearing capacity due to CMA decreases when the volume of FRP increases and the proposed method allows to quantify this effect;

(3) The load bearing capacity of FRP-strengthened RC beams with the consideration of CMA increases with increasing modulus of easticity of FRP and the influence of the FRP failure strain on CMA is negligible.

Acknowledgements

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